Digital System Design

Lecture 9

Karnaugh Map Methods- I

Objectives:

1. **Karnaugh Map definition.**
2. **Karnaugh map construction.**
    - **Two variables K-maps.**
    - **Three variables K-maps.**
    - **Four variables K-maps.**
3. **Summary**

1) Karnaugh map definition

- The karnaugh map (k-map) is a ***graphical tool*** used to simplify (minimize) the logic functions, so that it can be implemented with minimum number of gates (minimum number of product terms and minimum number of literals).
- The K-map is used to ***convert*** a truth table to its corresponding logic circuit.

2) Karnaugh map construction

- **Two variables K-maps.**
    - A two- variable function has four possible minterms, we can rearrange these minterms into a karnaugh map:

x	y	Minterms
0	0	$\bar{x}\bar{y}$
0	1	$\bar{x}y$
1	0	$x\bar{y}$
1	1	xy

x \ y	$(\bar{y})$ 0	(y) 1
0 $(\bar{x})$	$\bar{x}\,\bar{y}$	$\bar{x}\,y$
1 (x)	$x\,\bar{y}$	xy

⇒

x \ y	$(\bar{y})$ 0	(y) 1
0 $(\bar{x})$	m0	m1
1 (x)	m2	m3

Example:

$F = xy + x\bar{y}$

- The Karnaugh map for this function will be:

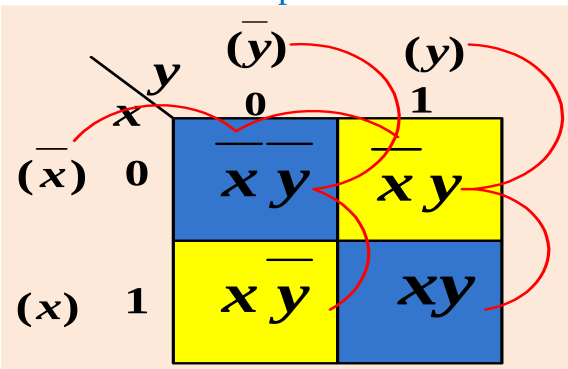
x \ y	$(\bar{y})$ 0	(y) 1
0 $(\bar{x})$	0	0
1 (x)	1	1

⇒ $F = xy + x\bar{y}$

Note: we can easily from the k-map see which ***minterms contain common literal***:

- Minterms on the left and right sides contain $\bar{y}$ and y respectively.

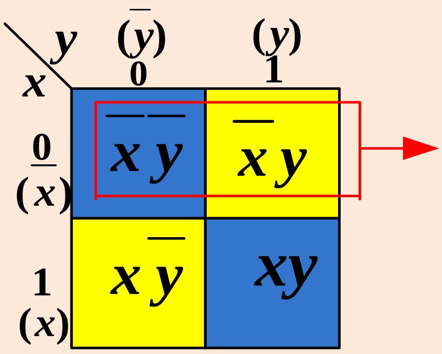
- Minterms in the top and bottom rows contain $\overline{x}$ and x respectively.



Note: each case in the truth table corresponds to a square in the K-map

- **Karnaugh map simplification:**
  - The K-map squares labeled so that ***horizontally adjacent*** square differ only in ***one variable.***

Example 1:- Imagine a two-variable sum of minterms, both of these minterms appear in the top row of a karnaugh map, which means that they both contain the literal $\overline{x}$.

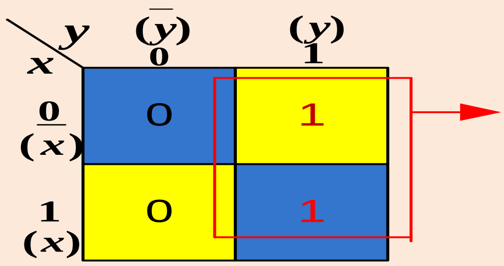


$$F = \overline{x}\,\overline{y} + \overline{x}y = \overline{x}(\overline{y} + y) = x$$

Example 2: $f = \overline{x}\,y + xy$ ***–minimize it using K-map.***

Solution:

- Both minterms appear in the right- side where y is uncomplemented.
- Thus, we can reduce $\overline{x}\,y + xy$ = just to y.

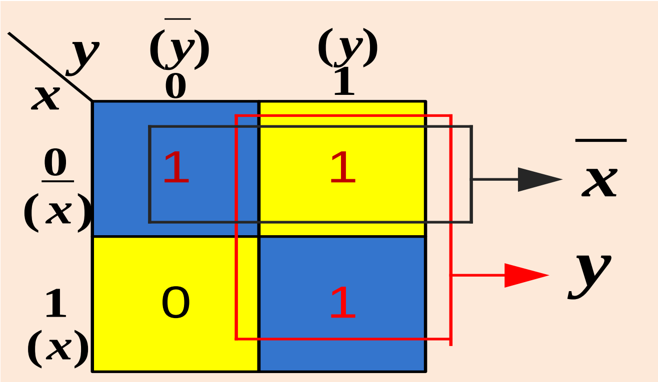


$$F = y$$

$$f = \overline{x}\,y + xy = y(x + \overline{x}) = y$$

Example 3: $z = \bar{x}\,\bar{y} + \bar{x}\,y + xy$ ***–minimize it using K-map.***

Solution:

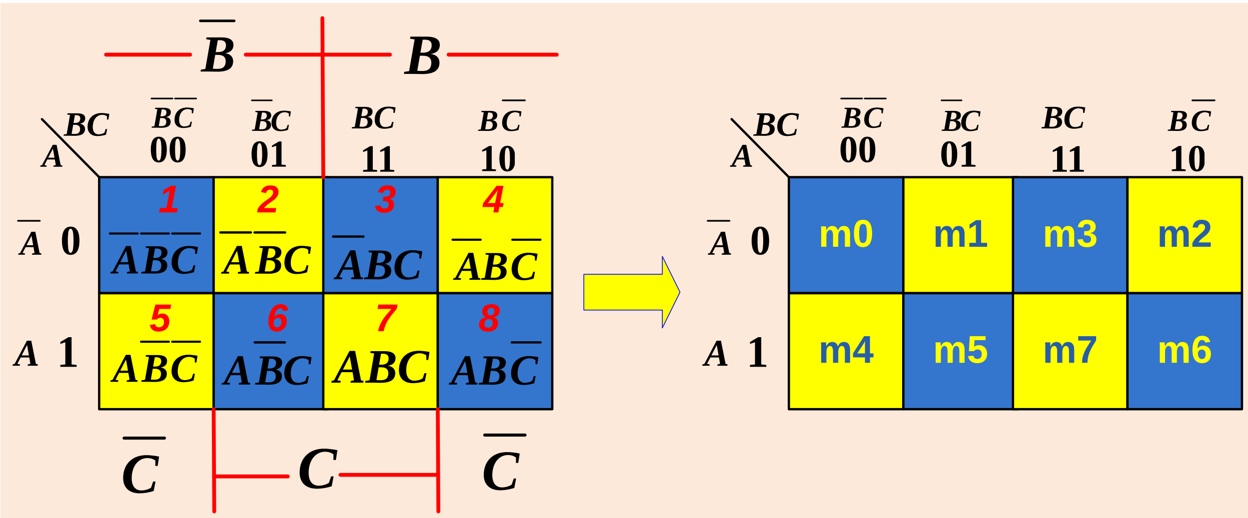


- ➢ We have $\bar{x}\,\bar{y} + \bar{x}y$ in the top row, corresponding to $\bar{x}$
- ➢ There's also $\bar{x}\,y + xy$ in the right side corresponding to y.
- ➢ The result $z = \bar{x} + y$.

Using algebraic simplification:

$$z = \bar{x}\,\bar{y} + \bar{x}\,y + xy = \bar{x}\,(\bar{y} + y) + xy$$
$$= \bar{x} + xy = (x + \bar{x})\,(\bar{x} + y) = \bar{x} + y$$

- **Three- variables K-map**
  - ➢ For $F(A, B, C)$ there are $2^3 = 8$ minterms.
  - ➢ ***Representation*** truth table using K-map
    - ○ ***Different versions:***



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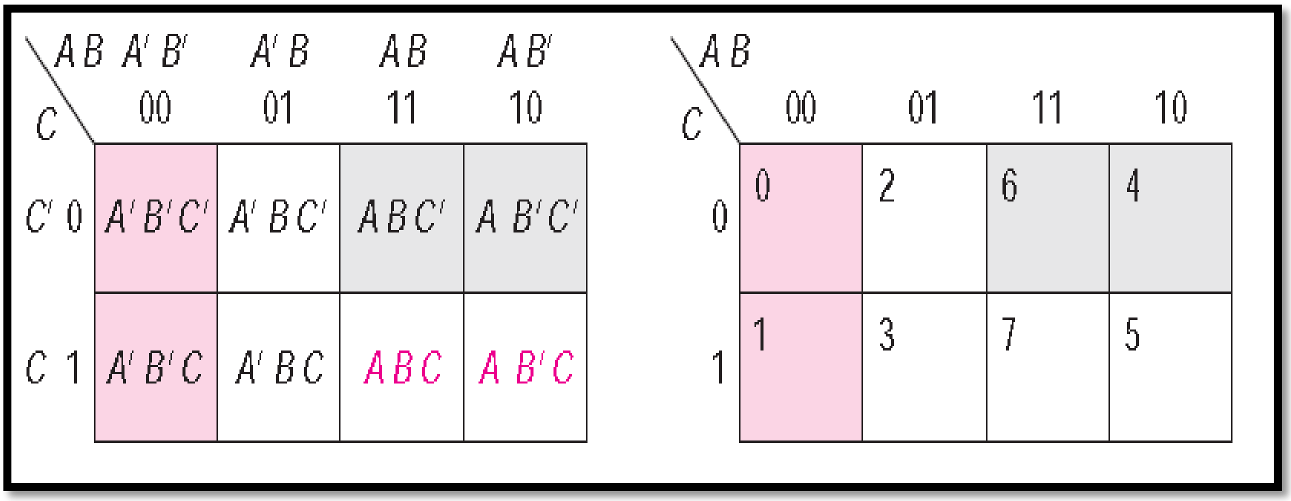

Grouping (ordering, looping)

- The groups can be ***2, 4, or 8 adjacent squares***:
  - ✓ ***2 squares*** ⟶ 1 variable can be canceled.
  - ✓ ***4 squares*** ⟶ 2 variables can be canceled.
  - ✓ ***8 squares*** ⟶ 3 variables can be canceled.

- ***Examples***:-

Squares	*Common Literal (s)*
(1) and (2)	$\bar{A}\,\bar{B}$
(2) and (3)	$\bar{A}C$
(1) and (4)	$\bar{A}\,\bar{C}$
(1), (2), (5) and (6)	$\bar{B}$
(3), (4), (7) and (8)	B
(1), (2), (3) and (4)	$\bar{A}$
(5), (6), (7) and (8)	A
(1), (5), (4) and (8) ***(Wrapping case is also adjacent)***	$\bar{C}$

- To ***proof*** the wrapping case (the last one in the table) algebraically, we can write:

$$F = \bar{A}\,\bar{B}\,\bar{C} + A\,\bar{B}\,\bar{C} + \bar{A}\,B\,\bar{C} + A\,B\,\bar{C}$$
$$= \bar{C}(\bar{A}\,\bar{B} + A\bar{B} + \bar{A}\,B + AB)$$
$$= \bar{C}\,(\bar{B}\,(\bar{A} + A) + B\,(\bar{A} + A))$$
$$= \bar{C}\,(\bar{B} + B) = \bar{C}$$

- "***Adjacency***" includes wrapping around the left and right side.

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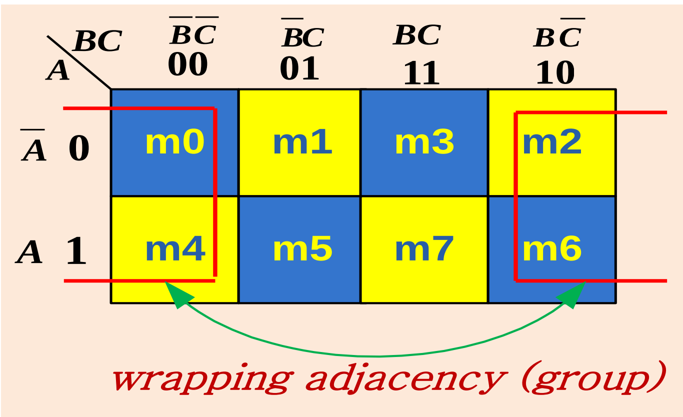


Example 1: ***Simplify the following logical function using K-map.***

$$F(x,y,z) = x\,y + \bar{y}\,z + x\,z$$

Solution:

Step 1: the ***expression must be in a sum of minterms form***, so we should convert it:(two ways to do that):

1. *Using logical rules (algebraically).*

$$F = x\,y + \bar{y}\,z + x\,z$$
$$= x\,z\,(z + \bar{z}) + \bar{y}\,z(x + \bar{x}) + x\,z\,(y + \bar{y})$$
$$= x\,y\,z + x\,y\,\bar{z} + x\,\bar{y}\,z + \bar{x}\,\bar{y}\,z + \cancel{x\,y}\,z + \cancel{x\,\bar{y}}\,z$$
$$= x\,y\,z + x\,y\,\bar{z} + x\,\bar{y}\,z + \bar{x}\,\bar{y}\,z$$
$$= m_1 + m_5 + m_6 + m_7$$

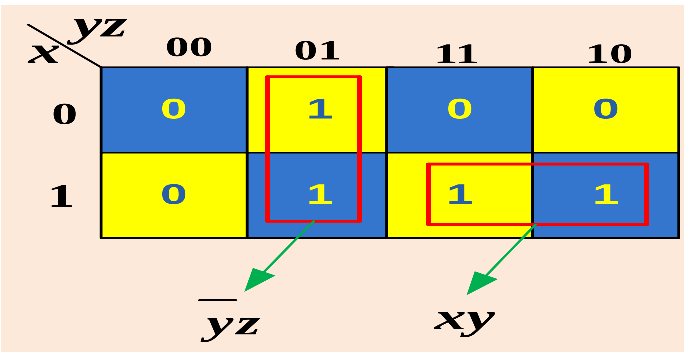
2. *Make the truth table and read the minterms.*

$$F = x\,y + \bar{y}\,z + x\,z$$

inputs			Output	Terms replacement
x	y	z	$F(x,y,z)$	
0	0	0	0	
0	0	1	1	$\bar{y}z$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	xz and $\bar{y}z$
1	1	0	1	xy
1	1	1	1	xy and xz

$$F(x,y,z) = \bar{x}\bar{y}z + x\,\bar{y}\,z + x\,y\,\bar{z} + x\,y\,z$$
$$= m_1 + m_5 + m_6 + m_7$$

Step 2: ***fill one's (for the minterms) in karnaugh map; zero's for other squares***.



Step 3: ***grouping*** (looping):

2 groups: ***(m1) and (m5)***
(m6) and (m7)

Step 4: ***simplify***:

$$F = x\,y + \bar{y}\,z$$

- To ***proof*** the result:

$$F = \bar{x}\bar{y}z + x\bar{y}z + xyz + xy\bar{z}$$
$$= \bar{y}z(x + \bar{x}) + xy(z + \bar{z})$$
$$= \bar{y}z + xy$$

Grouping the minterms:-

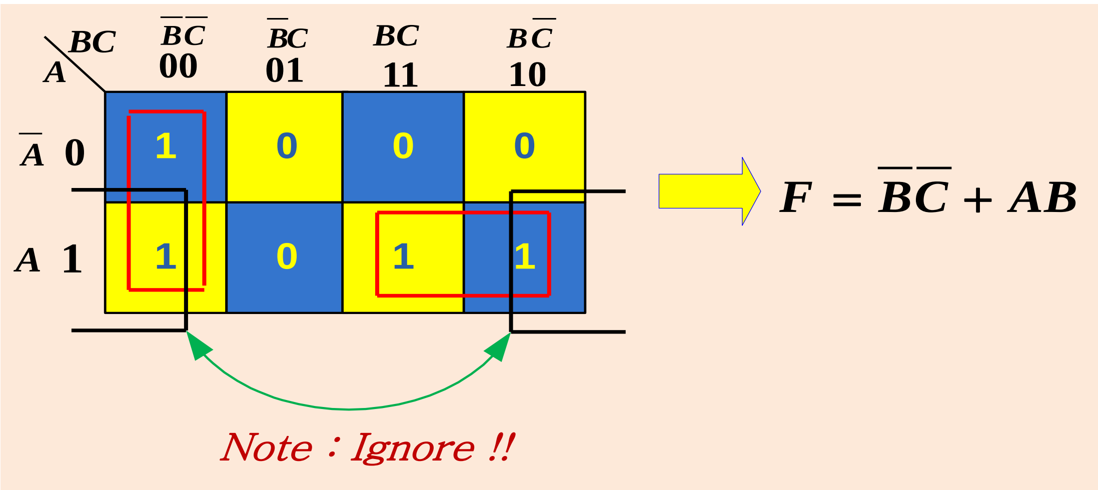
- Grouping together all the $\mathbf{1}_s$ in the K-map.
  - ***Make rectangles*** of 2^n (1, 2, 4, …).
  - All the $\mathbf{1}_s$ in the map should be included in at least one rectangle.
  - ***Do not*** include any of the $\mathbf{0}_s$.
  - Each group corresponds to one product term.
  - Make each rectangle ***as large as possible***.
  - We can ***overlap the rectangles***, if that makes them lager.

Example 2: ***Simplify the following logical function using K-map.***

$$Z = AB + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

Solution:

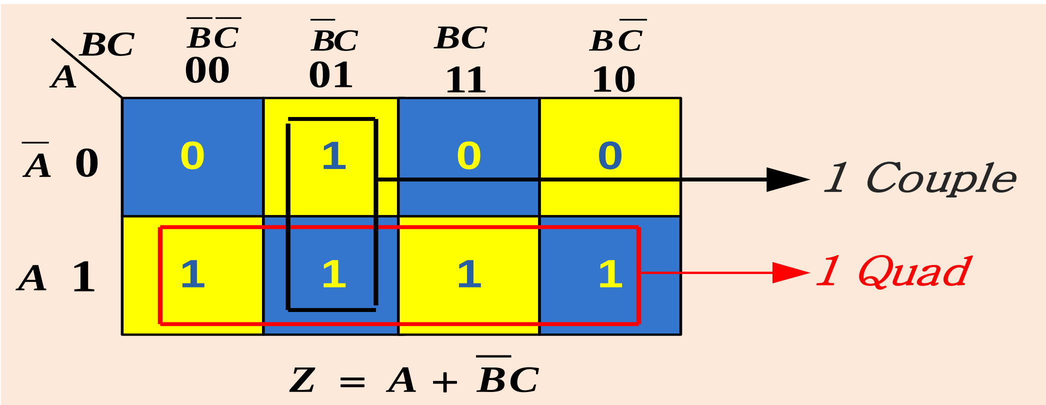
$$Z = AB + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} = AB\bar{C} + ABC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$



Example 3: ***Simplify the following logical function using K-map.***

$$Z = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

Solution:



Quad:

$$A\overline{B}C + A\overline{B}\,\overline{C} + AB\overline{C} + ABC = A(\overline{B}C + \overline{B}\,\overline{C} + B\overline{C} + BC)$$

$$= A(\overline{B}(C + \overline{C}) + B(\overline{C} + C)) = A(\overline{B} + B) = A$$

Couple:

$$\overline{A}\overline{B}C + A\overline{B}C = \overline{B}C(A + A) = \overline{B}C$$